TM5 Problem 3 – 24

For β = 0.2 s^{-1}, produce computer plots like those shown in Figure 3-15 for a sinusoidal driven, damped oscillator where $x_p(t)$, $x_c(t)$ and the sum *x(t)* are shown. Let *k = 1 kg/*s^2 and *m = 1 kg*. Do this for values of ω_D/ω_S of *1/9*, *1/3*, *1.1*, *3* and *6*. For the $x_c(t)$ solution (Eqn. 3.40, the underdamped case), let the phase angle *δ = 0*. And the amplitude *A = -1 m*. For the $x_p(t)$ solution (Eqn. 3.60), let F_0/m= *1 m/*s^2, but calculate *δ*. What do you observe about the relative amplitudes of the two solutions as ω_D increases? Why does this occur? For ω_D/ω_S = *6*, let *A = 20 m/*s^2 for $x_p(t)$ and produce the plot again.

In[1]:=
```
xc[t_] := A * Exp[-β * t] * Cos[ωS * t - θ]
xp[t_] := B * Cos[ωD * t - ϕ]
```

$$B = \frac{\frac{F_0}{m}}{\sqrt{\left(\omega_N^2 - \omega_D^2\right)^2 + 4\beta^2\omega_D^2}}$$

$$\phi = \text{ArcTan}\left[\frac{2 * \beta * \omega_D}{\omega_N^2 - \omega_D^2}\right]$$

Out[3]= $$\frac{F_0}{m\sqrt{4\beta^2\omega_D^2 + (-\omega_D^2 + \omega_N^2)^2}}$$

Out[4]= $$\text{ArcTan}\left[\frac{2\beta\omega_D}{-\omega_D^2 + \omega_N^2}\right]$$

Giving

In[6]:=
```
x[t_] = xc[t] + xp[t]
```

Out[6]= $$A e^{-t\beta}\text{Cos}[\theta - t\omega_S] + \frac{\text{Cos}\left[\text{ArcTan}\left[\frac{2\beta\omega_D}{-\omega_D^2+\omega_N^2}\right] - t\omega_D\right]F_0}{m\sqrt{4\beta^2\omega_D^2 + (-\omega_D^2 + \omega_N^2)^2}}$$

Taking values of

In[6]:=
```
θ = 0;
m = 1;
k = 1;
β = 0.2;
A = -1;
F0 = 1;
```

$$\omega_N = \sqrt{\frac{k}{m}};$$

$$\omega_S = \sqrt{\omega_N^2 - \beta^2}$$

Out[13]= 0.979796

Which gives $\omega_S = 0.979796\ s^{-1}$. This gives

TM5 Problem 3 – 24

For β = 0.2 s^{-1}, produce computer plots like those shown in Figure 3-15 for a sinusoidal driven, damped oscillator where $x_p(t)$, $x_c(t)$ and the sum $x(t)$ are shown. Let $k = 1\ kg/s^2$ and $m = 1\ kg$. Do this for values of ω_D/ω_S of 1/9, 1/3, 1.1, 3 and 6. For the $x_c(t)$ solution (Eqn. 3.40, the underdamped case), let the phase angle $\delta = 0$. And the amplitude $A = -1\ m$. For the $x_p(t)$ solution (Eqn. 3.60), let F_0/m= $1\ m/s^2$, but calculate δ. What do you observe about the relative amplitudes of the two solutions as ω_D increases? Why does this occur? For $\omega_D/\omega_S = 6$, let $A = 20\ m/s^2$ for $x_p(t)$ and produce the plot again.

In[1]:=
```
xc[t_] := A * Exp[-β * t] * Cos[ωS * t - θ]
xp[t_] := B * Cos[ωD * t - ϕ]
```
$$B = \frac{\frac{F_0}{m}}{\sqrt{\left(\omega_N^2 - \omega_D^2\right)^2 + 4\beta^2\omega_D^2}}$$

$$\phi = \text{ArcTan}\left[\frac{2 * \beta * \omega_D}{\omega_N^2 - \omega_D^2}\right]$$

Out[3]= $$\frac{F_0}{m\sqrt{4\beta^2\omega_D^2 + \left(-\omega_D^2 + \omega_N^2\right)^2}}$$

Out[4]= $$\text{ArcTan}\left[\frac{2\beta\omega_D}{-\omega_D^2 + \omega_N^2}\right]$$

Giving

In[6]:=
```
x[t_] = xc[t] + xp[t]
```

Out[6]= $$A\, e^{-t\beta}\,\text{Cos}[\theta - t\,\omega_S] + \frac{\text{Cos}\left[\text{ArcTan}\left[\frac{2\beta\omega_D}{-\omega_D^2+\omega_N^2}\right] - t\,\omega_D\right] F_0}{m\sqrt{4\beta^2\omega_D^2 + \left(-\omega_D^2 + \omega_N^2\right)^2}}$$

Taking values of

In[6]:=
```
θ = 0;
m = 1;
k = 1;
β = 0.2;
A = -1;
F0 = 1;
```
$$\omega_N = \sqrt{\frac{k}{m}};$$

$$\omega_S = \sqrt{\omega_N^2 - \beta^2}$$

Out[13]= 0.979796

Which gives $\omega_S = 0.979796\ s^{-1}$. This gives

```
In[14]:= Expand[xc[t]]
         Expand[xp[t]]
         Expand[x[t]]
```

Out[14]= $-e^{-0.2\,t}\,\text{Cos}[0.979796\,t]$

Out[15]= $$\frac{\text{Cos}\left[\text{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+(1-\omega_D^2)^2}}$$

Out[16]= $$-e^{-0.2\,t}\,\text{Cos}[0.979796\,t]+\frac{\text{Cos}\left[\text{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+(1-\omega_D^2)^2}}$$

To make the plots, first take $\frac{\omega_D}{\omega_S}=\frac{1}{9}$ so that $\omega_D=\frac{\omega_S}{9}$, or

In[17]:= ω_S

$\omega_D = \frac{\omega_S}{9}$

```
Expand[x[t]]
```

Out[17]= 0.979796

Out[18]= 0.108866

Out[19]= $1.01101\,\text{Cos}[0.0440403-0.108866\,t]-e^{-0.2\,t}\,\text{Cos}[0.979796\,t]$

Plot for $\frac{\omega_D}{\omega_S}=\frac{1}{9}$

```
In[20]:= pxc1 = Plot[xc[t], {t, 0, 35}, BaseStyle →
            {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
          PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
           {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"]
         pxp1 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-2, 2},
          PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
          PlotLabel → "Steady State"]
         px1 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
            FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-2, 2}, PlotPoints → 100,
          PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"]
         px1color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
           PlotPoints → 100, PlotStyle → {{RGBColor[0, 0.5, 0]}}];
         Show[pxc1, pxp1, px1, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
            \(D\)]\) = (1/9)\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

```
In[14]:= Expand[xc[t]]
         Expand[xp[t]]
         Expand[x[t]]
```

Out[14]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]$

Out[15]= $$\frac{\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+(1-\omega_D^2)^2}}$$

Out[16]= $$-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]+\frac{\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+(1-\omega_D^2)^2}}$$

To make the plots, first take $\frac{\omega_D}{\omega_S}=\frac{1}{9}$ so that $\omega_D=\frac{\omega_S}{9}$, or

```
In[17]:= ωS
         ωD = ωS/9
         Expand[x[t]]
```

Out[17]= 0.979796

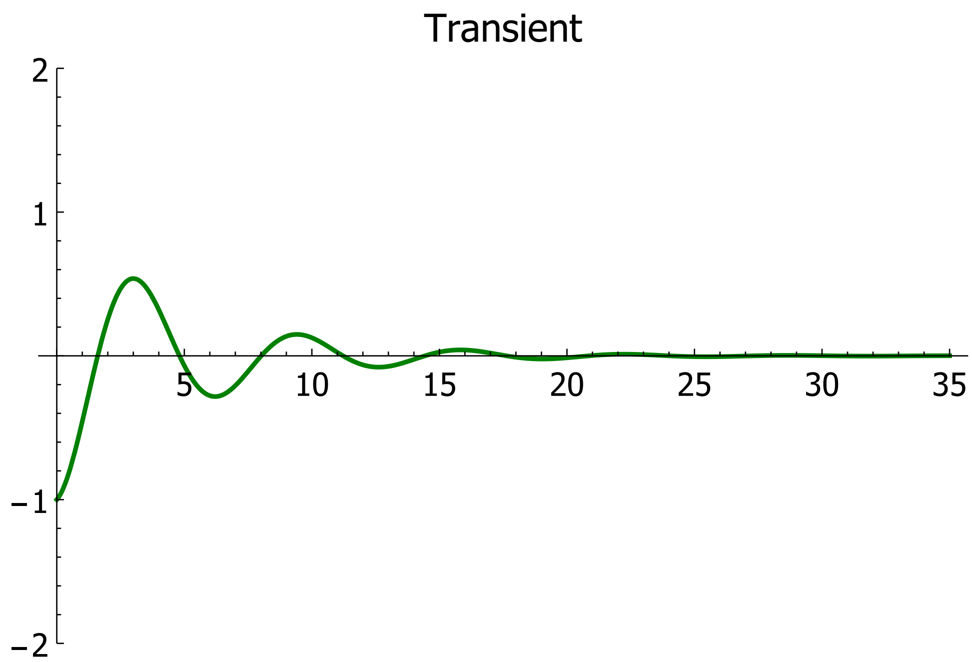
Out[18]= 0.108866

Out[19]= $1.01101\,\mathrm{Cos}[0.0440403-0.108866\,t]-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]$

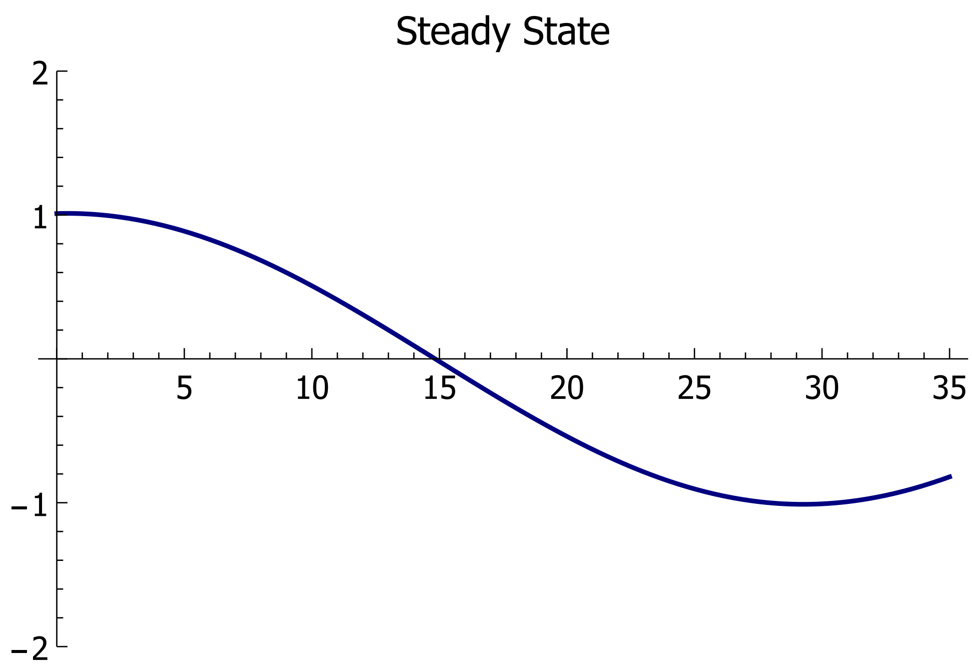
Plot for $\frac{\omega_D}{\omega_S}=\frac{1}{9}$

```
In[20]:= pxc1 = Plot[xc[t], {t, 0, 35}, BaseStyle →
            {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
          PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
           {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"]
         pxp1 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-2, 2},
          PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
          PlotLabel → "Steady State"]
         px1 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
            FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-2, 2}, PlotPoints → 100,
          PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"]
         px1color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
           PlotPoints → 100, PlotStyle → {{RGBColor[0, 0.5, 0]}}];
         Show[pxc1, pxp1, px1, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
            \(D\)]\) = (1/9)\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

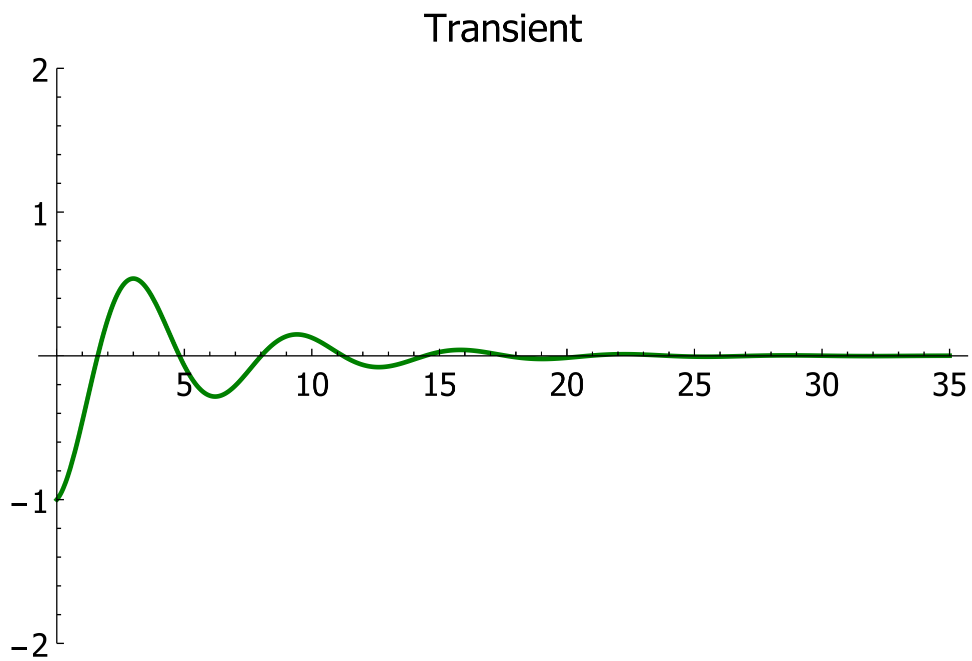
Out[20]=


Transient
2
1
-1
-2
5
10
15
20
25
30
35

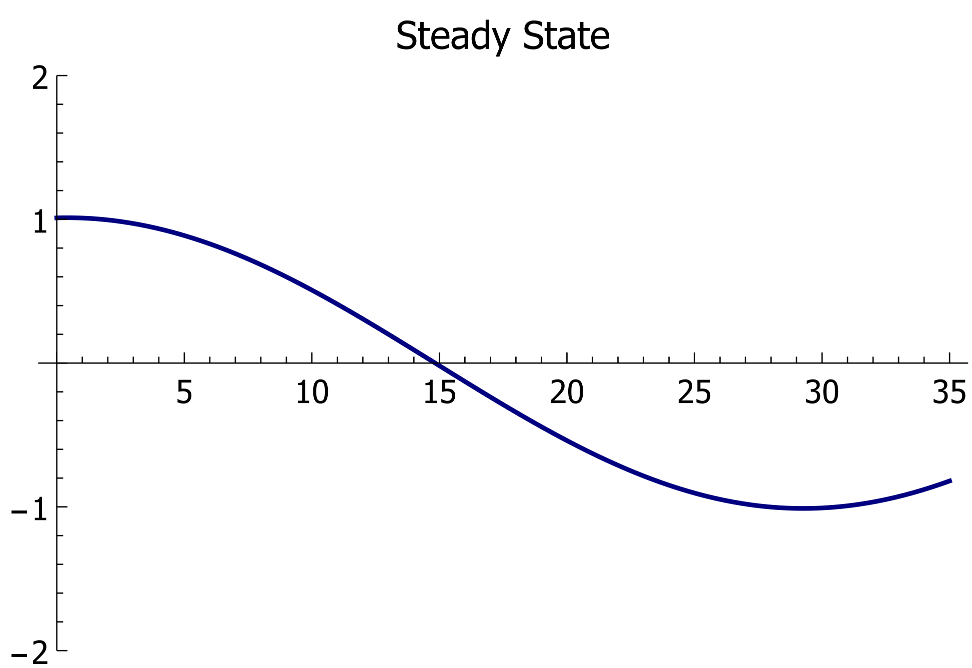
Out[21]=


Steady State
2
1
-1
-2
5
10
15
20
25
30
35

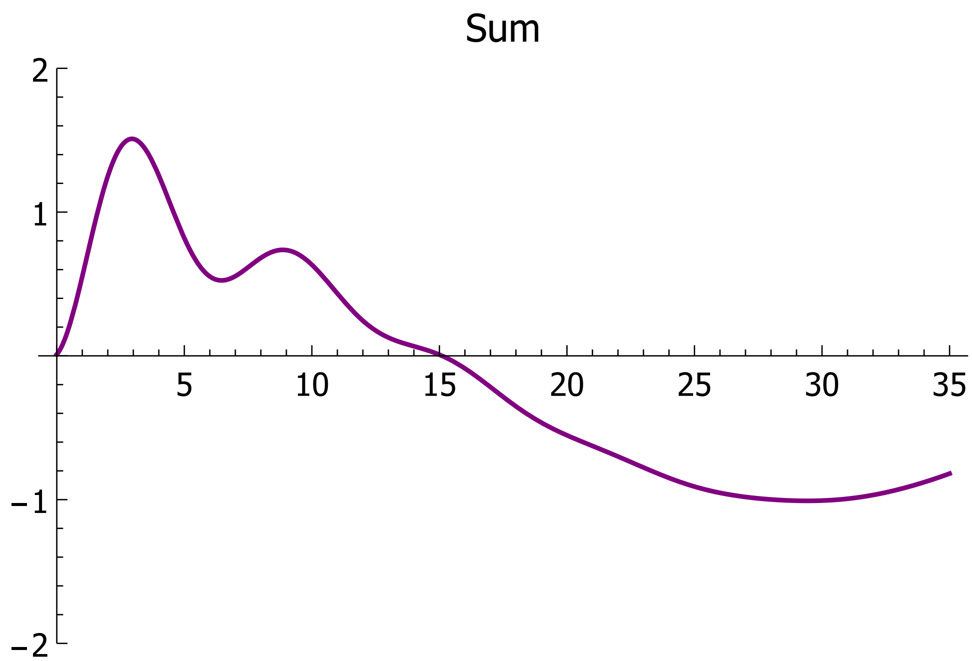
Out[20]=


Transient
2
1
5
10
15
20
25
30
35
−1
−2

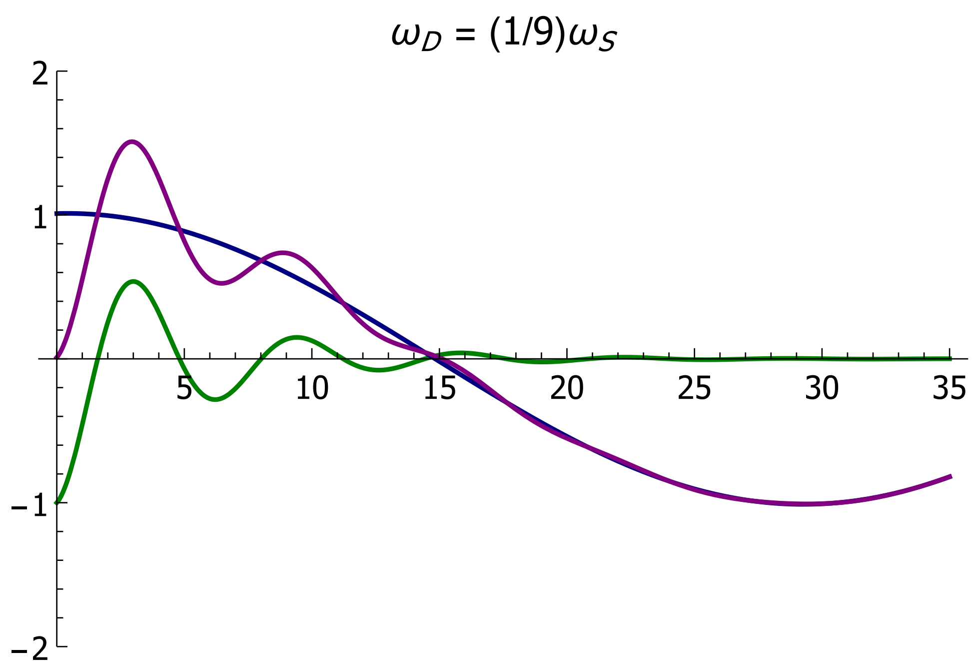
Out[21]=


Steady State
2
1
5
10
15
20
25
30
35
−1
−2

Out[22]=



Out[24]=



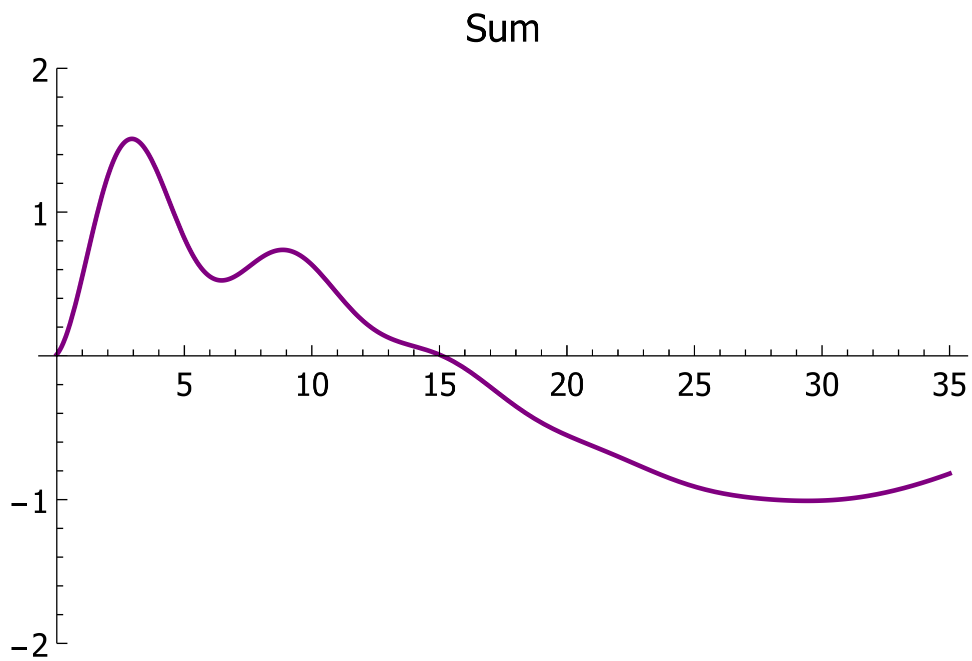
Now take $\frac{\omega_D}{\omega_S} = \frac{1}{3}$ so that $\omega_D = \frac{\omega_S}{3}$, or

In[25]:=
```
ωS
ωD = ωS/3
Expand[x[t]]
```

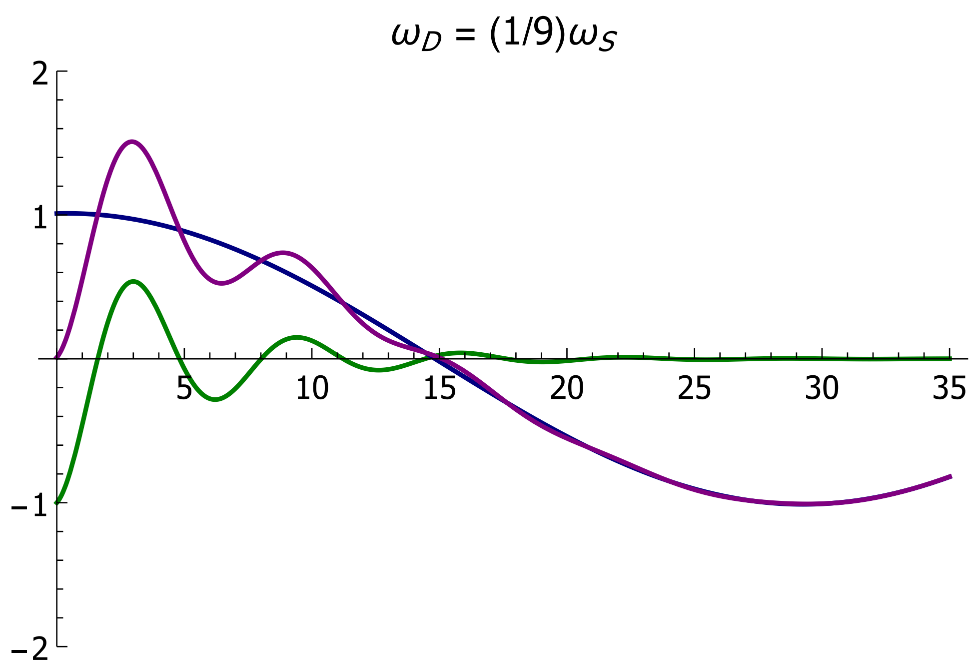
Out[25]= 0.979796

Out[26]= 0.326599

Out[27]= $1.10762 \operatorname{Cos}[0.145209 - 0.326599\, t] - e^{-0.2\, t} \operatorname{Cos}[0.979796\, t]$



Out[22]=



Out[24]=

Now take $\frac{\omega_D}{\omega_S} = \frac{1}{3}$ so that $\omega_D = \frac{\omega_S}{3}$, or

In[25]:= ω_S

$\omega_D = \frac{\omega_S}{3}$

Expand[x[t]]

Out[25]= 0.979796

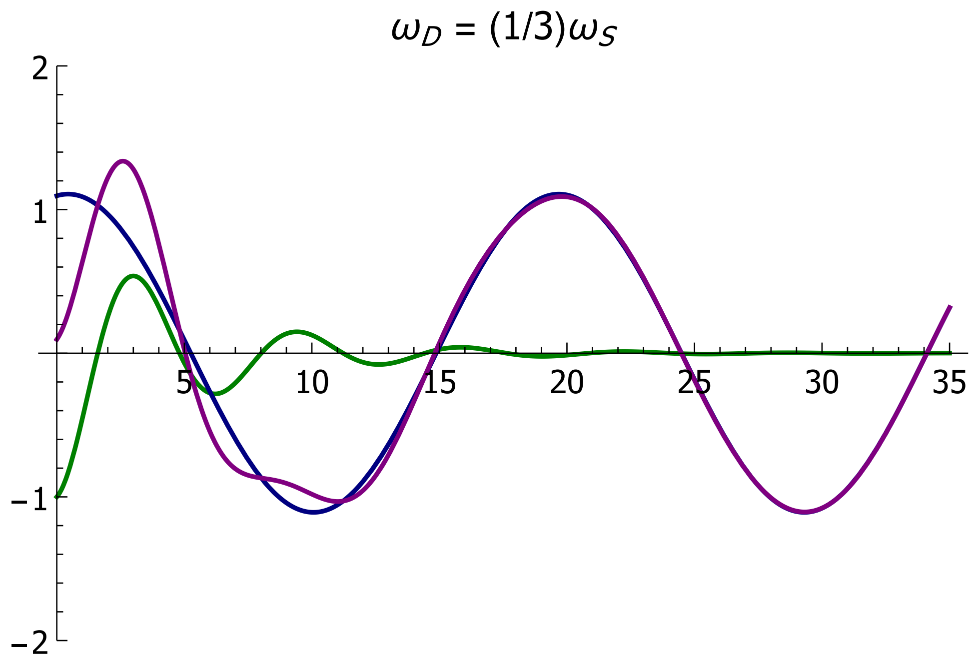
Out[26]= 0.326599

Out[27]= $1.10762\,\text{Cos}[0.145209 - 0.326599\,t] - e^{-0.2\,t}\,\text{Cos}[0.979796\,t]$

Plot for $\frac{\omega_D}{\omega_S} = \frac{1}{3}$

In[28]:=
```
pxc2 = Plot[xc[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
   PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
pxp2 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-2, 2},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
   PlotLabel → "Steady State"];
px2 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
     FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-2, 2}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
px2color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[0.5, 0.5, 0]}}];
Show[pxc2, pxp2, px2, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = (1/3)\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[32]=

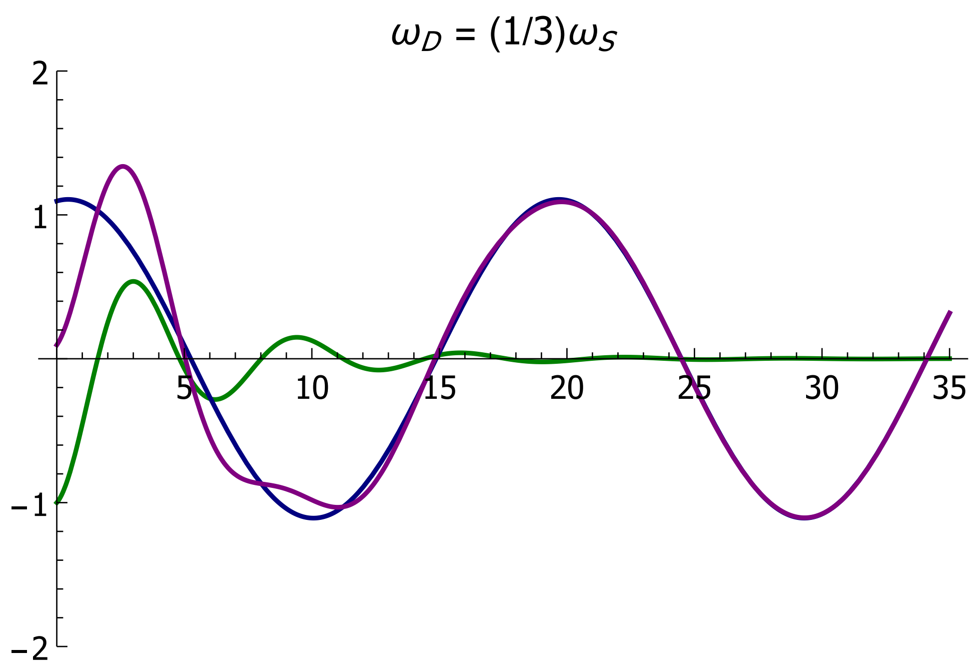


Next take $\frac{\omega_D}{\omega_S} = 1.1$ so that $\omega_D = 1.1\ \omega_S$, or

Plot for $\frac{\omega_D}{\omega_S} = \frac{1}{3}$

In[28]:=
```
pxc2 = Plot[xc[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
   PlotRange → {-2, 2}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
pxp2 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-2, 2},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
   PlotLabel → "Steady State"];
px2 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
     FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-2, 2}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
px2color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[0.5, 0.5, 0]}}];
Show[pxc2, pxp2, px2, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = (1/3)\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[32]=



Next take $\frac{\omega_D}{\omega_S} = 1.1$ so that $\omega_D = 1.1\ \omega_S$, or

In[58]:=
```
ωS
ωD = 1.1 * ωS
Expand[x[t]]
```

Out[58]= 0.979796

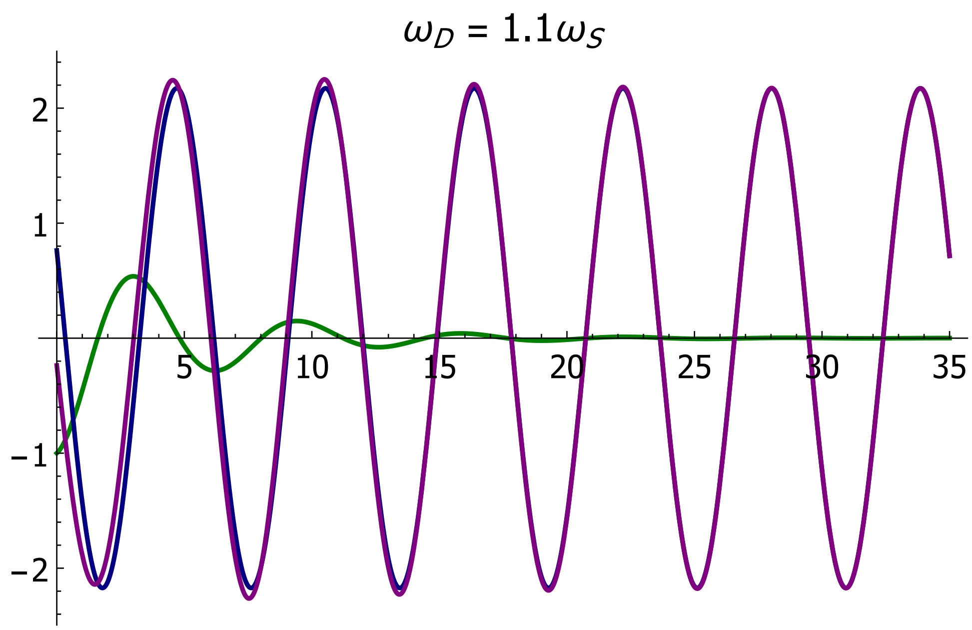
Out[59]= 1.07778

Out[60]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t] + 2.17201\,\mathrm{Cos}[1.21216 + 1.07778\,t]$

Plot for $\frac{\omega_D}{\omega_S} = 1.1$

In[61]:=
```
pxc3 = Plot[xc[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
   PlotRange → {-2.5, 2.5}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
pxp3 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
   PlotLabel → "Steady State"];
px3 = Plot[x[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0.5, 0, 0.5]},
   PlotRange → {-2.5, 2.5}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
px3color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 1]}}];
Show[pxc3, pxp3, px3, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = 1.1\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[65]=



In[58]:=
```
ωS
ωD = 1.1 * ωS
Expand[x[t]]
```

Out[58]= 0.979796

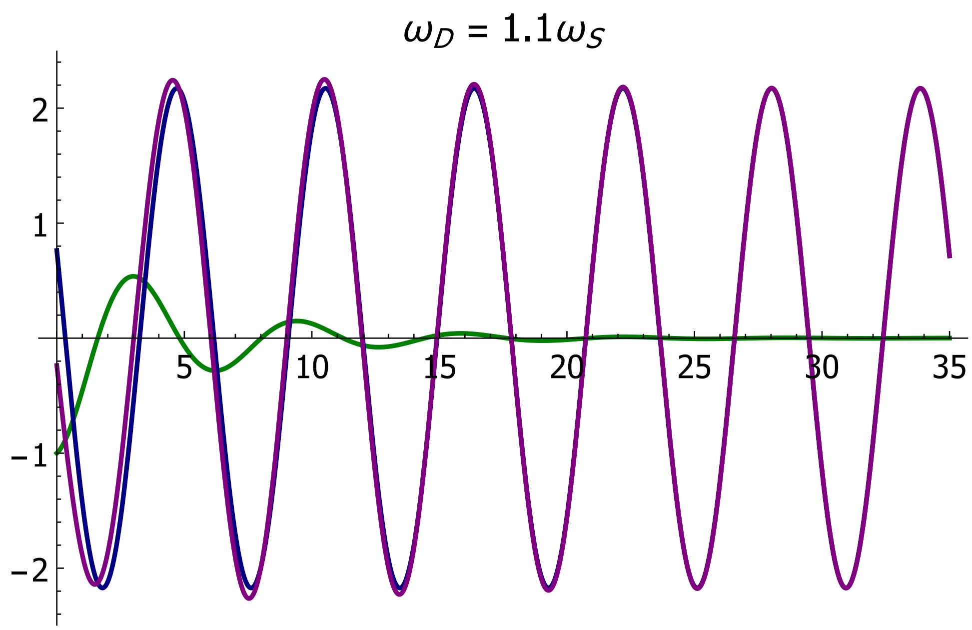
Out[59]= 1.07778

Out[60]= $-e^{-0.2\,t}\,\text{Cos}[0.979796\,t] + 2.17201\,\text{Cos}[1.21216 + 1.07778\,t]$

Plot for $\frac{\omega_D}{\omega_S} = 1.1$

In[61]:=
```
pxc3 = Plot[xc[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
   PlotRange → {-2.5, 2.5}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
pxp3 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
   PlotLabel → "Steady State"];
px3 = Plot[x[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0.5, 0, 0.5]},
   PlotRange → {-2.5, 2.5}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
px3color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 1]}}];
Show[pxc3, pxp3, px3, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = 1.1\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[65]=



Next take $\frac{\omega_D}{\omega_S} = 3$ so that $\omega_D = 3\,\omega_S$, or

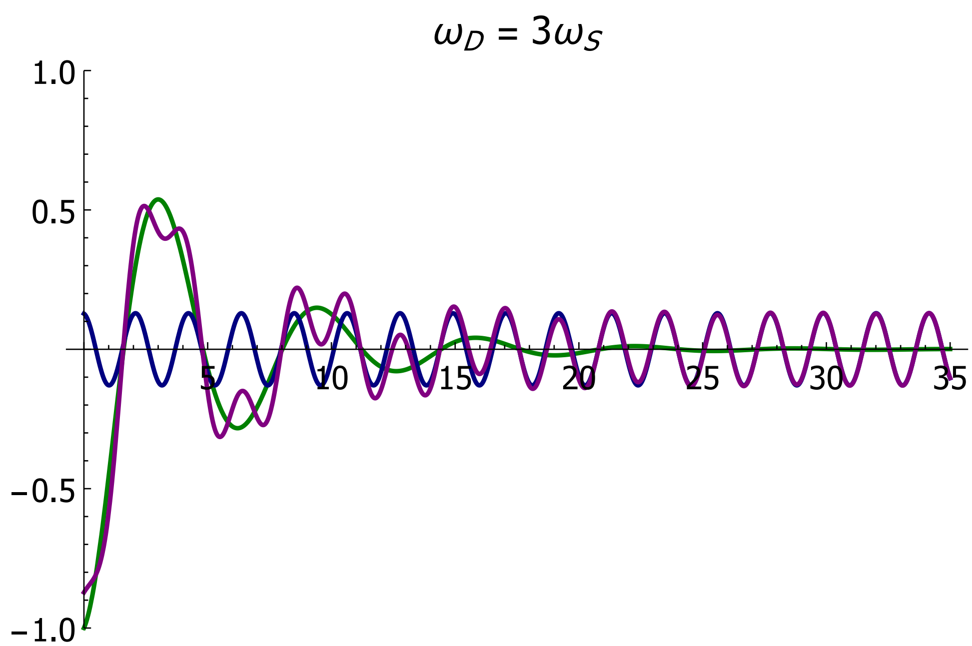
```
In[41]:= ωS
         ωD = 3 * ωS
         Expand[x[t]]
Out[41]= 0.979796
Out[42]= 2.93939
```

Out[43]= $-e^{-0.2\,t}\,\text{Cos}[0.979796\,t] + 0.129367\,\text{Cos}[0.152697 + 2.93939\,t]$

Plot for $\frac{\omega_D}{\omega_S} = 3$

```
In[44]:= pxc4 = Plot[xc[t], {t, 0, 35}, BaseStyle →
           {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
          PlotRange → {-1, 1}, PlotPoints → 100, PlotStyle →
           {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
         pxp4 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-1, 1},
          PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
          PlotLabel → "Steady State"];
         px4 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
            FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-1, 1}, PlotPoints → 100,
          PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
         px4color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
          PlotPoints → 100, PlotStyle → {{RGBColor[0.5, 0, 0.5]}}];
         Show[pxc4, pxp4, px4, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
           \(D\)]\) = 3\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[48]=



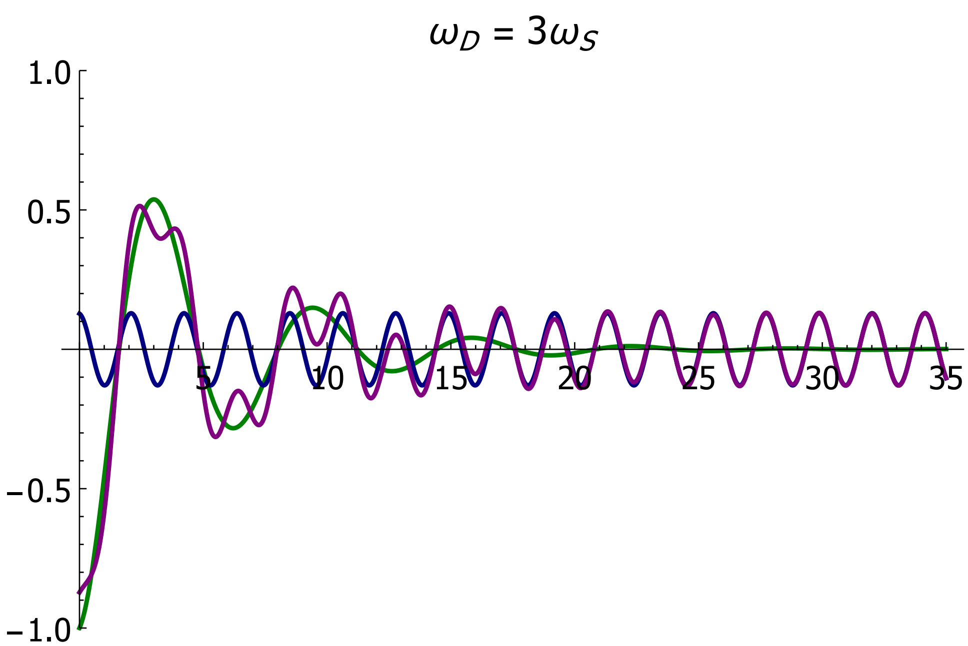
Next take $\frac{\omega_D}{\omega_S} = 3$ so that $\omega_D = 3\,\omega_S$, or

```
In[41]:= ωS
         ωD = 3 * ωS
         Expand[x[t]]
Out[41]= 0.979796
Out[42]= 2.93939
Out[43]= -e^(-0.2 t) Cos[0.979796 t] + 0.129367 Cos[0.152697 + 2.93939 t]
```

Plot for $\frac{\omega_D}{\omega_S} = 3$

```
In[44]:= pxc4 = Plot[xc[t], {t, 0, 35}, BaseStyle →
           {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
          PlotRange → {-1, 1}, PlotPoints → 100, PlotStyle →
           {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
         pxp4 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-1, 1},
          PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
          PlotLabel → "Steady State"];
         px4 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
            FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-1, 1}, PlotPoints → 100,
          PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
         px4color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
          PlotPoints → 100, PlotStyle → {{RGBColor[0.5, 0, 0.5]}}];
         Show[pxc4, pxp4, px4, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
           \(D\)]\) = 3\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[48]=



Next take $\frac{\omega_D}{\omega_S} = 6$ so that $\omega_D = 6\,\omega_S$, or

In[67]:=
```
ωS
ωD = 6 * ωS
Expand[x[t]]
```

Out[67]= 0.979796

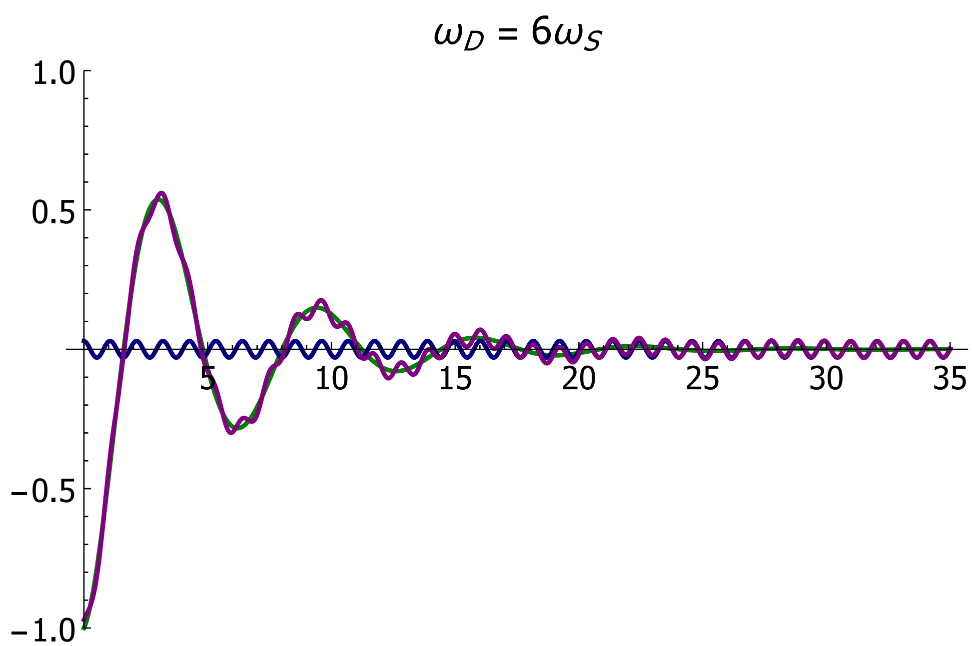
Out[68]= 5.87878

Out[69]= $-e^{-0.2\,t}\,\text{Cos}[0.979796\,t] + 0.0297245\,\text{Cos}[0.0699545 + 5.87878\,t]$

Plot for $\frac{\omega_D}{\omega_S} = 6$

In[70]:=
```
pxc5 = Plot[xc[t], {t, 0, 35}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
   PlotRange → {-1, 1}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
pxp5 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-1, 1},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
   PlotLabel → "Steady State"];
px5 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
     FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-1, 1}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
px5color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
   PlotPoints → 100, PlotStyle → {{RGBColor[1, 0, 0]}}];
Show[pxc5, pxp5, px5, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = 6\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[74]=



The amplitude of the steady-state term can be seen to increase as ω_D approaches the value of ω_S then decrease as ω_D gets much larger than the value of ω_S. This is due to the dependence of the

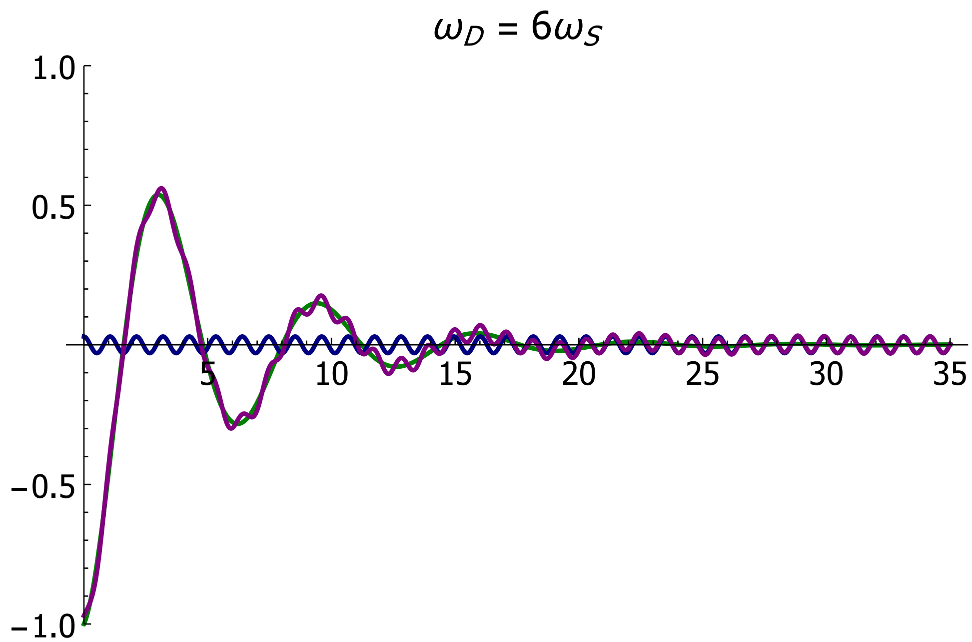
Next take $\frac{\omega_D}{\omega_S} = 6$ so that $\omega_D = 6\,\omega_S$, or

```
In[67]:= ωS
         ωD = 6 * ωS
         Expand[x[t]]
Out[67]= 0.979796
Out[68]= 5.87878
Out[69]= -e^(-0.2 t) Cos[0.979796 t] + 0.0297245 Cos[0.0699545 + 5.87878 t]
```

Plot for $\frac{\omega_D}{\omega_S} = 6$

```
In[70]:= pxc5 = Plot[xc[t], {t, 0, 35}, BaseStyle →
           {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
          PlotRange → {-1, 1}, PlotPoints → 100, PlotStyle →
           {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"];
         pxp5 = Plot[xp[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica,
            FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-1, 1},
          PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
          PlotLabel → "Steady State"];
         px5 = Plot[x[t], {t, 0, 35}, BaseStyle → {FontFamily → Helvetica, FontSize → 12,
            FontColor → RGBColor[0.5, 0, 0.5]}, PlotRange → {-1, 1}, PlotPoints → 100,
          PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"];
         px5color = Plot[x[t], {t, 0, 35}, PlotRange → {-2.5, 2.5},
          PlotPoints → 100, PlotStyle → {{RGBColor[1, 0, 0]}}];
         Show[pxc5, pxp5, px5, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
           \(D\)]\) = 6\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[74]=



The amplitude of the steady-state term can be seen to increase as ω_D approaches the value of ω_S then decrease as ω_D gets much larger than the value of ω_S. This is due to the dependence of the steady-state amplitude on the frequencies

$$H = \frac{\frac{F_0}{m}}{\sqrt{\left(\omega_N{}^2-\omega_D{}^2\right)^2+4\,\beta^2\,\omega_D{}^2}}.$$

We can factor out ω_D to show this dependence:

$$H = \frac{\frac{F_0}{m}}{\omega_D\sqrt{\left(\frac{\omega_N{}^2}{\omega_D{}^2}-1\right)^2+4\,\beta^2}}.$$

This shows that as the steady-state frequency increases, its amplitude decreases.

For ω_D/ω_S = 6, let A = 20 m/s^2 (this is F_0 in my expression) for x_p(t) and produce the plot again. Also reduce the number of cycles because it gets too crowded.

Taking values of

In[7]:=
```
θ = 0;
m = 1;
k = 1;
β = 0.2;
A = -1;
B = 1;
F0 = 20;
ωN = Sqrt[k/m];
ωS = Sqrt[ωN^2 - β^2];
```

In[16]:=
```
Expand[xc[t]]
Expand[xp[t]]
Expand[x[t]]
```

Out[16]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]$

Out[17]= $\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]$

Out[18]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]+\frac{20\,\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right]-t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2+\left(1-\omega_D^2\right)^2}}$

Plot this for ω_D = 6 ω_S, or steady-state amplitude on the frequencies

$$H = \frac{\frac{F_0}{m}}{\sqrt{\left(\omega_N{}^2 - \omega_D{}^2\right)^2 + 4\,\beta^2\,\omega_D{}^2}}.$$

We can factor out ω_D to show this dependence:

$$H = \frac{\frac{F_0}{m}}{\omega_D \sqrt{\left(\frac{\omega_N{}^2}{\omega_D{}^2} - 1\right)^2 + 4\,\beta^2}}.$$

This shows that as the steady-state frequency increases, its amplitude decreases.

For ω_D/ω_S = 6, let A = 20 m/s^2 (this is F_0 in my expression) for x_p(t) and produce the plot again. Also reduce the number of cycles because it gets too crowded.

Taking values of

```
In[7]:= θ = 0;
        m = 1;
        k = 1;
        β = 0.2;
        A = -1;
        B = 1;
        F0 = 20;
        ωN = Sqrt[k/m];
        ωS = Sqrt[ωN^2 - β^2];
```

```
In[16]:= Expand[xc[t]]
         Expand[xp[t]]
         Expand[x[t]]
```

Out[16]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t]$

Out[17]= $\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right] - t\,\omega_D\right]$

Out[18]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t] + \frac{20\,\mathrm{Cos}\left[\mathrm{ArcTan}\left[\frac{0.4\,\omega_D}{1-\omega_D^2}\right] - t\,\omega_D\right]}{\sqrt{0.16\,\omega_D^2 + (1-\omega_D^2)^2}}$

Plot this for $\omega_D = 6\,\omega_S$, or

In[19]:=
```
ωS
ωD = 6 * ωS
Expand[x[t]]
```

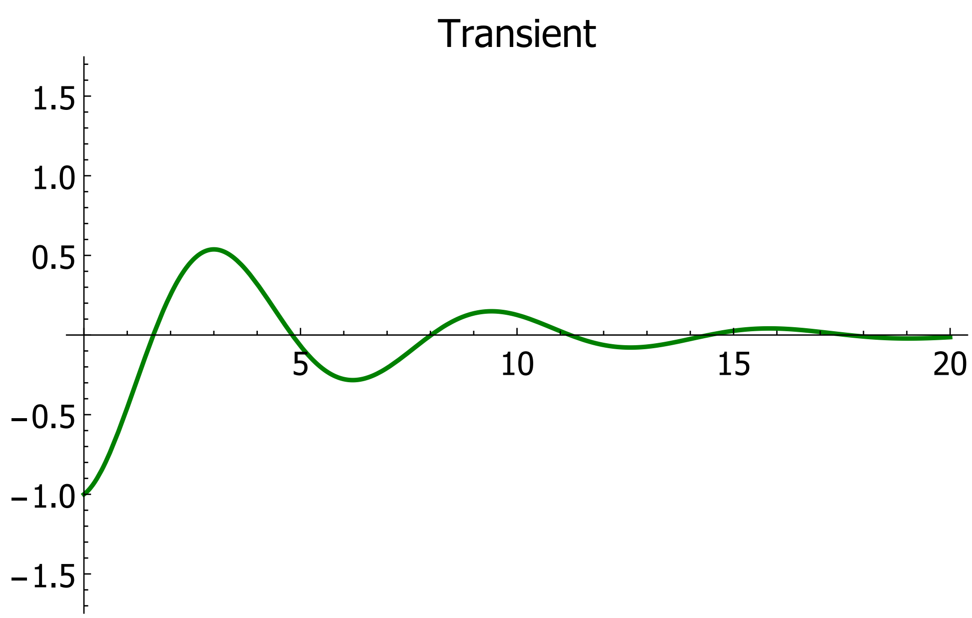
Out[19]= 0.979796

Out[20]= 5.87878

Out[21]= $-e^{-0.2\,t}\,\mathrm{Cos}[0.979796\,t] + 0.59449\,\mathrm{Cos}[0.0699545 + 5.87878\,t]$

In[22]:=
```
pxc6 = Plot[xc[t], {t, 0, 20}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
  PlotRange → {-1.75, 1.75}, PlotPoints → 100, PlotStyle →
   {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"]
pxp6 = Plot[xp[t], {t, 0, 20}, BaseStyle → {FontFamily → Helvetica,
    FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-1.75, 1.75},
  PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
  PlotLabel → "Steady State"]
px6 = Plot[x[t], {t, 0, 20}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0.5, 0, 0.5]},
  PlotRange → {-1.75, 1.75}, PlotPoints → 100,
  PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"]
Show[pxc6, pxp6, px6, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = 6\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

Out[22]=



In[19]:=
```
ωS
ωD = 6 * ωS
Expand[x[t]]
```

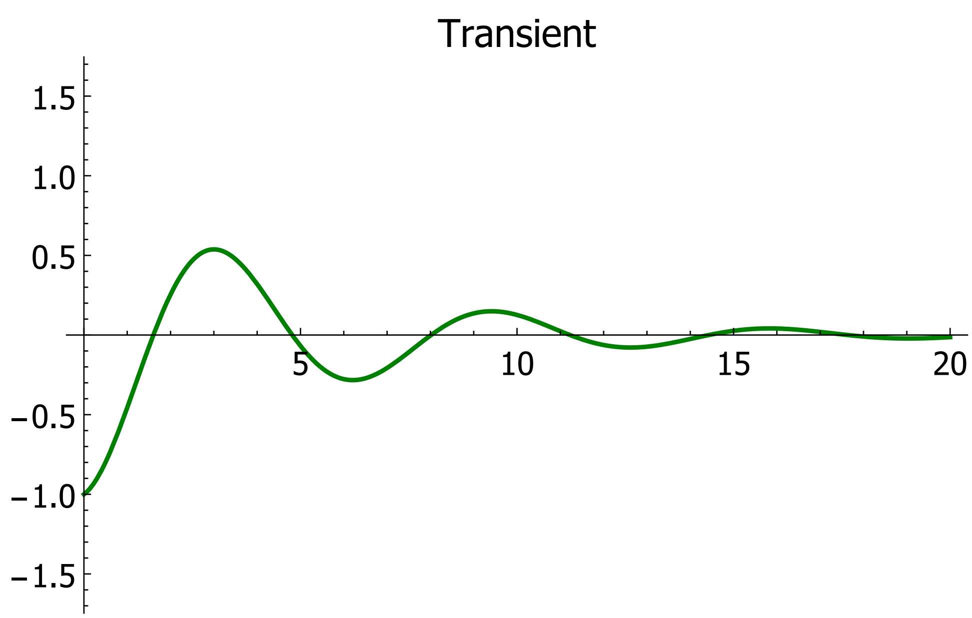
Out[19]= 0.979796

Out[20]= 5.87878

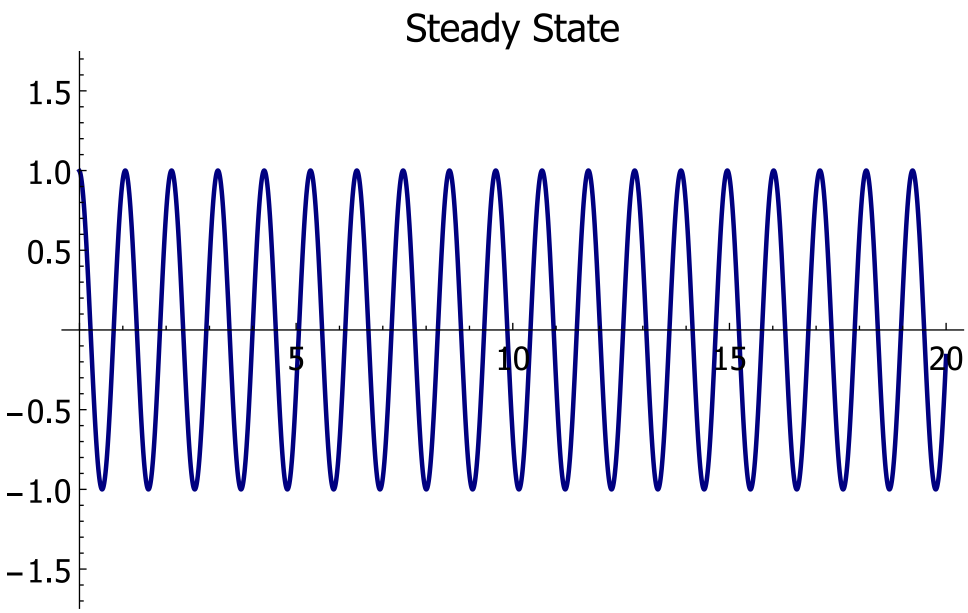
Out[21]= $-e^{-0.2\,t}\,\text{Cos}[0.979796\,t] + 0.59449\,\text{Cos}[0.0699545 + 5.87878\,t]$

In[22]:=
```
pxc6 = Plot[xc[t], {t, 0, 20}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
   PlotRange → {-1.75, 1.75}, PlotPoints → 100, PlotStyle →
    {{RGBColor[0, 0.5, 0], Thickness[0.005]}}, PlotLabel → "Transient"]
pxp6 = Plot[xp[t], {t, 0, 20}, BaseStyle → {FontFamily → Helvetica,
     FontSize → 12, FontColor → RGBColor[0, 0, 0.5]}, PlotRange → {-1.75, 1.75},
   PlotPoints → 100, PlotStyle → {{RGBColor[0, 0, 0.5], Thickness[0.005]}},
   PlotLabel → "Steady State"]
px6 = Plot[x[t], {t, 0, 20}, BaseStyle →
    {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0.5, 0, 0.5]},
   PlotRange → {-1.75, 1.75}, PlotPoints → 100,
   PlotStyle → {{RGBColor[0.5, 0, 0.5], Thickness[0.005]}}, PlotLabel → "Sum"]
Show[pxc6, pxp6, px6, PlotLabel → "\!\(\*SubscriptBox[\(ω\),
    \(D\)]\) = 6\!\(\*SubscriptBox[\(ω\), \(S\)]\)"]
```

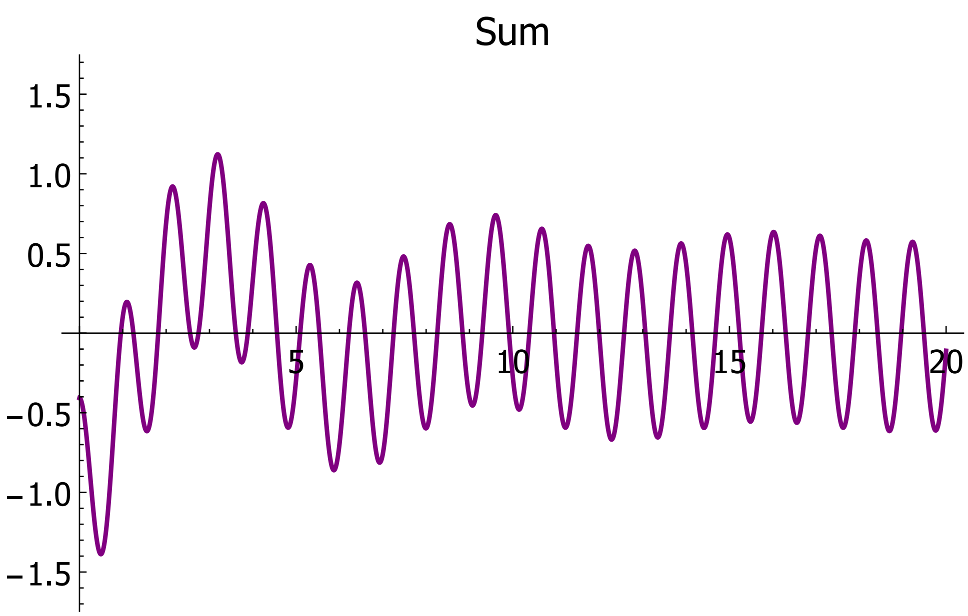
Out[22]=



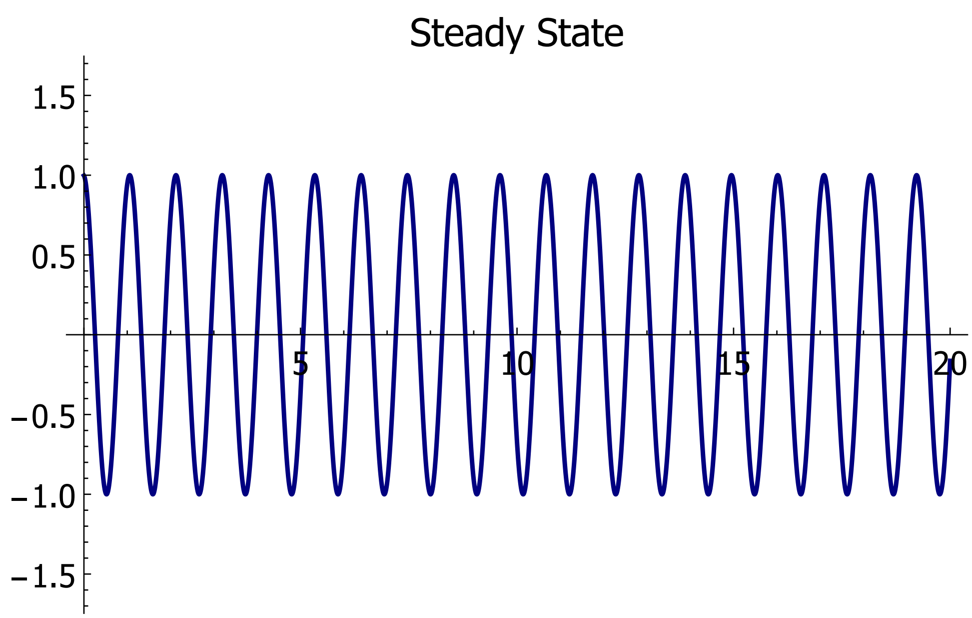
Out[23]=


Steady State
1.5
1.0
0.5
−0.5
−1.0
−1.5
5
10
15
20

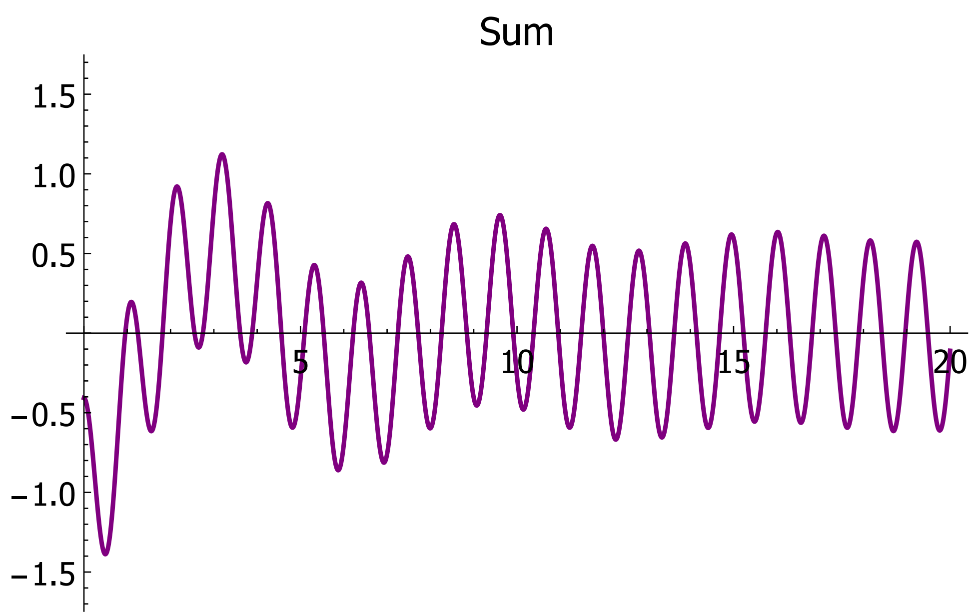
Out[24]=


Sum
1.5
1.0
0.5
−0.5
−1.0
−1.5
5
10
15
20

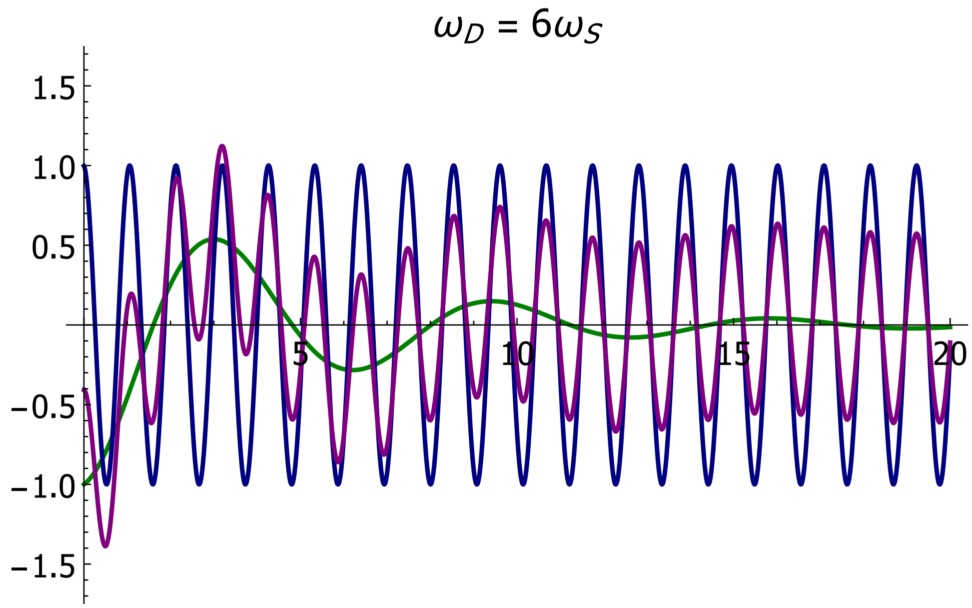
Out[23]=


Steady State
1.5
1.0
0.5
−0.5
−1.0
−1.5
5
10
15
20

Out[24]=


Sum
1.5
1.0
0.5
−0.5
−1.0
−1.5
5
10
15
20

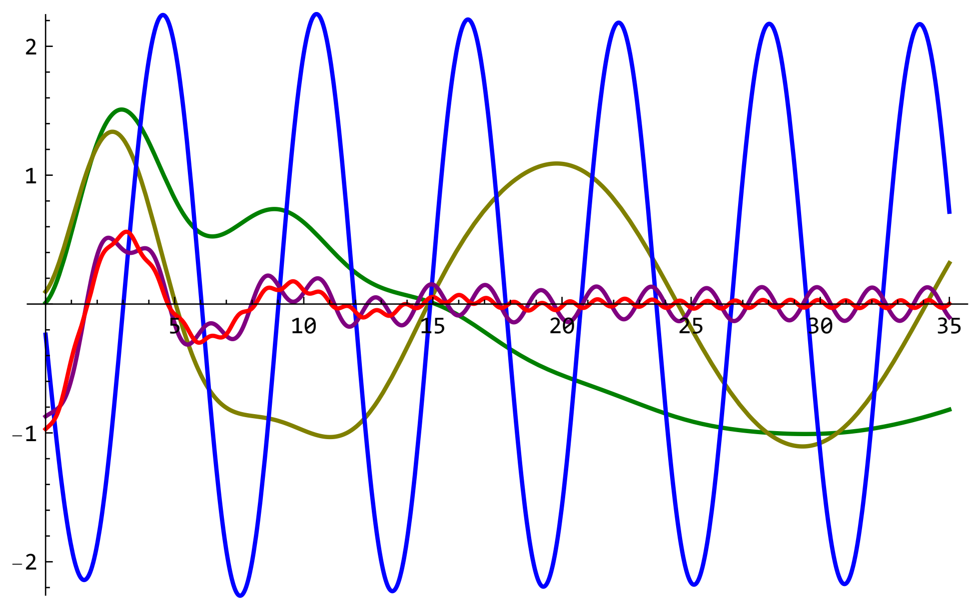
Out[25]=



Plot just the sums for ω_D/ω_S = 1/9, 1/3, 1.1, 3, and 6.

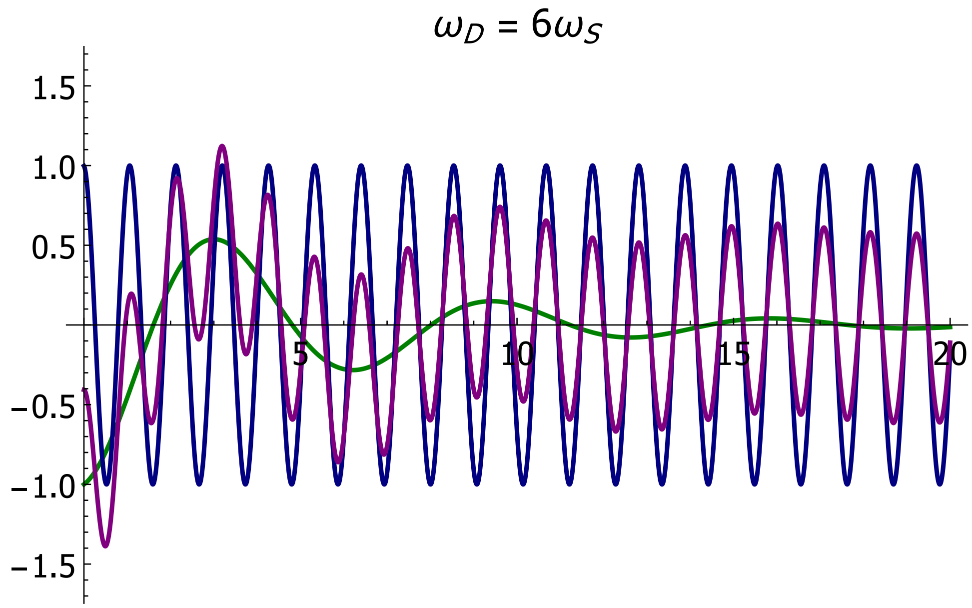
In[75]:=
```
Show[px1color, px2color, px3color, px4color, px5color, PlotRange → All]
```

Out[75]=



Since the amplitude of the steady-state function has $\sqrt{(\omega_S^2-\omega_D^2)^2-4\,\beta^2\,\omega_D^2}$ in the denominator, the amplitude of the steady state decreases as the difference between the frequencies increases ... the amplitude is greatest for the smallest difference, positive or negative! As for the warping, the first two (1/9 green, 1/3 olive) show the transient strongly warping the steady state. The last two (3 maroon, 6 red) show the steady state warping the transient. Both die out after about three system periods (τ_S=$2\pi/\omega_S$) The ω_D/ω_S = 1.1 plot in blue shows the transient having very little effect on the steady state. So the transient has the greatest effect on steady state functions of increasing frequency difference, greater or lesser. The last plot of ω_D/ω_S = 6 with a larger steady state

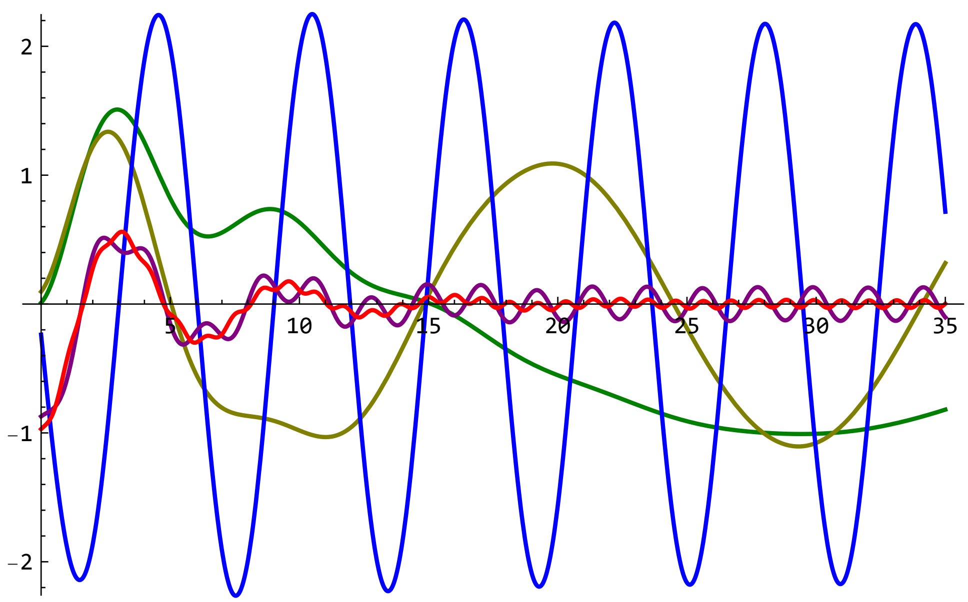
Out[25]=



Plot just the sums for ω_D/ω_S = 1/9, 1/3, 1.1, 3, and 6.

In[75]:=
```
Show[px1color, px2color, px3color, px4color, px5color, PlotRange → All]
```

Out[75]=



Since the amplitude of the steady-state function has $\sqrt{(\omega_S^2 - \omega_D^2)^2 - 4\,\beta^2\,\omega_D^2}$ in the denominator, the amplitude of the steady state decreases as the difference between the frequencies increases ... the amplitude is greatest for the smallest difference, positive or negative! As for the warping, the first two (1/9 green, 1/3 olive) show the transient strongly warping the steady state. The last two (3 maroon, 6 red) show the steady state warping the transient. Both die out after about three system periods (τ_S=$2\pi/\omega_S$) The ω_D/ω_S = 1.1 plot in blue shows the transient having very little effect on the steady state. So the transient has the greatest effect on steady state functions of increasing frequency difference, greater or lesser. The last plot of ω_D/ω_S = 6 with a larger steady state amplitude shows more strongly that the steady state warps the transient when it has a greater frequency.

```
In[1]:= Export["TM5Pr03_24Mathematica.pdf", SelectedNotebook[]]
Out[1]= TM5Pr03_24Mathematica.pdf
```

amplitude shows more strongly that the steady state warps the transient when it has a greater frequency.

```
In[1]:= Export["TM5Pr03_24Mathematica.pdf", SelectedNotebook[]]

Out[1]= TM5Pr03_24Mathematica.pdf
```